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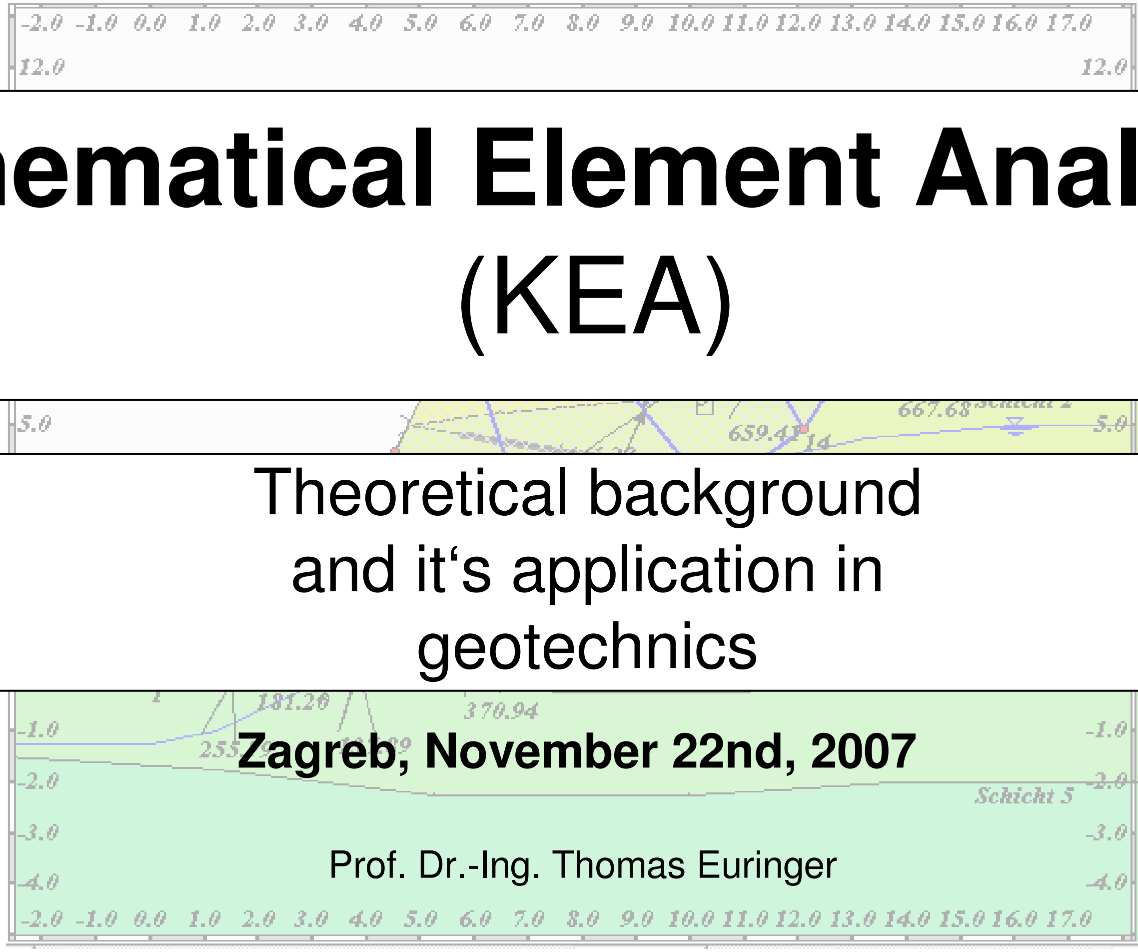


Kinematical Element Analysis (KEA)

Theoretical background and it‘s application in geotechnics

Zagreb, November 22nd, 2007

Prof. Dr.-Ing. Thomas Euringer

Outline

- **Concepts and fundamental principles**
- **Mechanical Model**
  - Modeling of multi-part wedge mechanisms
    - Solution of the system's kinematics and kinematic consistency
    - Statics
    - How to construct KE-systems properly
- **Optimization**
    - Topology …
    - Geometry of the system
- **Examples**
    - Stability of slopes
    - Stability of excavation pit constructions
    - Bearing capacity
    - How to take into consideration of flow

▪ Concepts and fundamental principles

KEA

Generalized version of rigid wedge analysis based on Coulomb‘s <u>ultimate limit state</u>

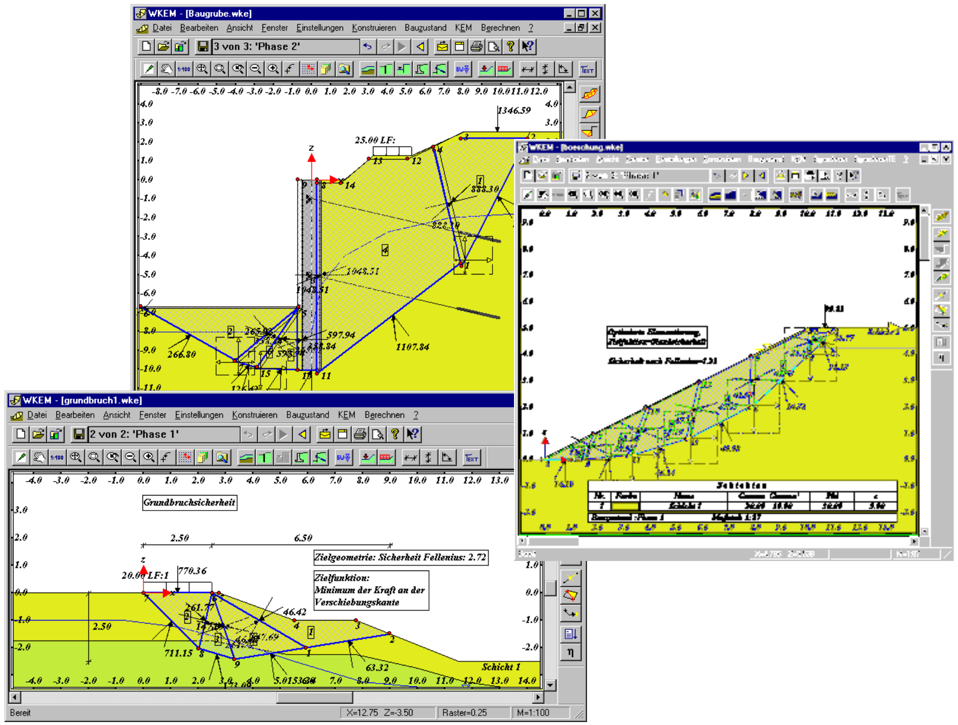
- Free description of the failure mechanism using rigid bodies
- Soil-to-soil contact due to Coulomb‘s equation
  $\sigma_t = (\sigma_n - u) \cdot \tan(\varphi) + c$
- Equilibrium of forces acc. to principle of virtual displacements
  $A_i + A_a = min$
- Optimization of the geometry

Applicability of KEA in geotechnics

Ultimate limit state analysis

Realistically modelling of failure mechanisms to calculate ….

- Internal / external stability
- Bearing capacity failure
- active/passive
  earth pressure



Input values ⟷ Results

- **Input values:** Soil parameters
  - φ ***(Internal friction)***
  - $\boldsymbol{c}$ ***(Cohesion)***
- **Results:**
  - Interacting forces at the element's boundaries
  - Stability acc. to Fellenius
    (*Reduction of shear parameters* $\varphi - \boldsymbol{c}$)
  - Geometry of failure mechanism
  - Displacement*rates*

Tensions, displacements do not get calculated

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Mechanical Model

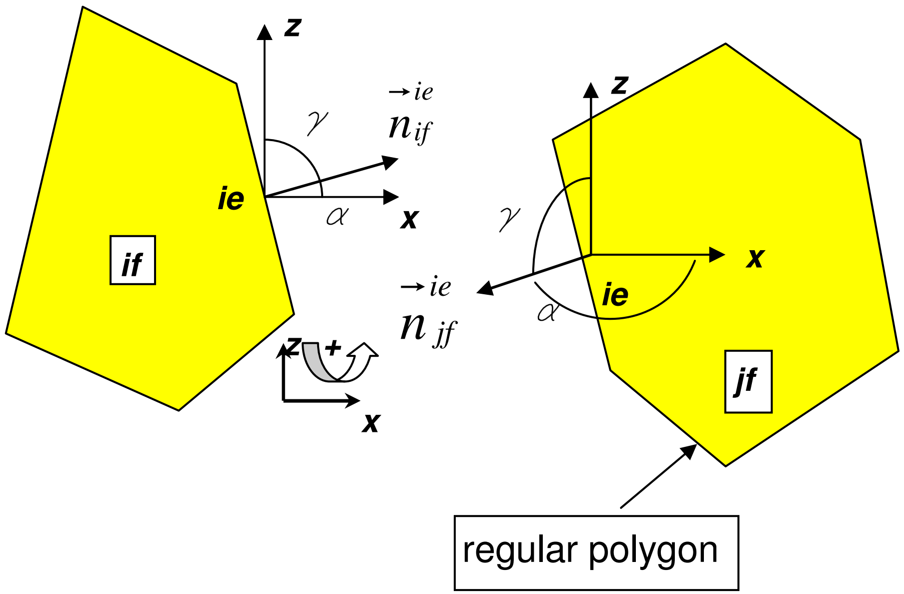
Geometry edge, face

$$n_{ie}^{if\,T} = [\cos\alpha \quad \cos\gamma]_{ie}^{if}$$

$$\cos\alpha_{ie}^{if} = \frac{dz_{ie}^{if}}{d_{ie}} \qquad \cos\gamma_{ie}^{if} = -\frac{dx_{ie}^{if}}{d_{ie}}$$

$$dx_{ie}^{if} = x_{jv}^{if} - x_{iv}^{if} \qquad dz_{ie}^{if} = z_{jv}^{if} - z_{iv}^{if}$$

$$d_{ie} = \sqrt{(dx_{ie}^{if})^2 + (dz_{ie}^{if})^2}$$



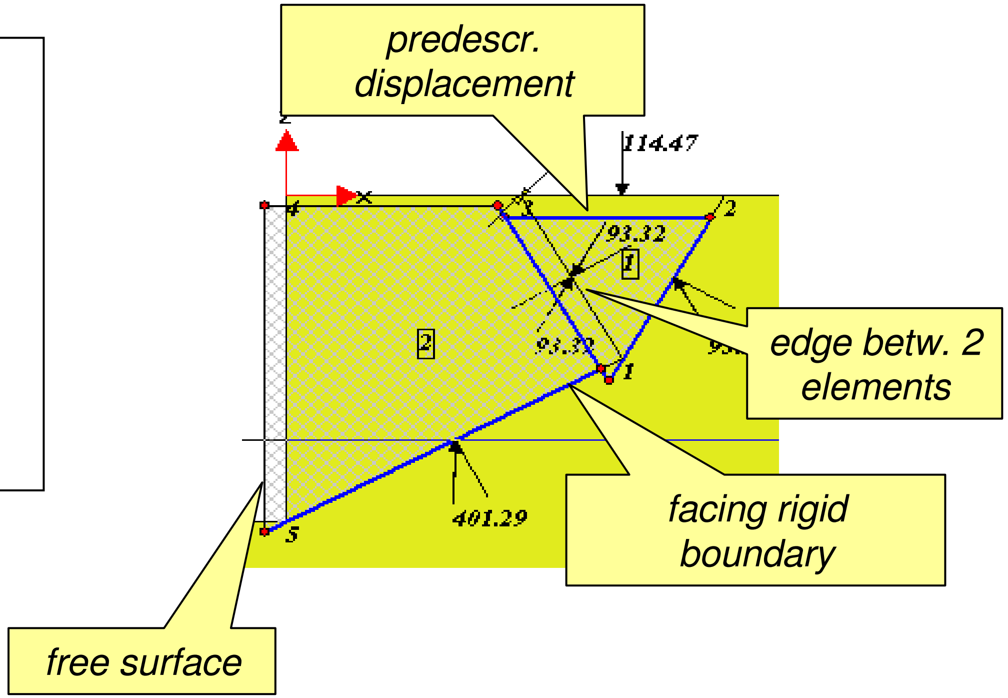
Kinematics

Multi-part wedge structure

Boundary conditions:

- **Edges between elements**
- **Edges with predescribed displacement**
- **Edges facing rigid boundary area**
- **Edges with no constraints (free resp. gaping edges)**

$$K^{sys} \cdot v^{sys} = r^{sys}$$



Kinematics: Example

Setting up system kinematic-matrix for a 2-wedge mechanism

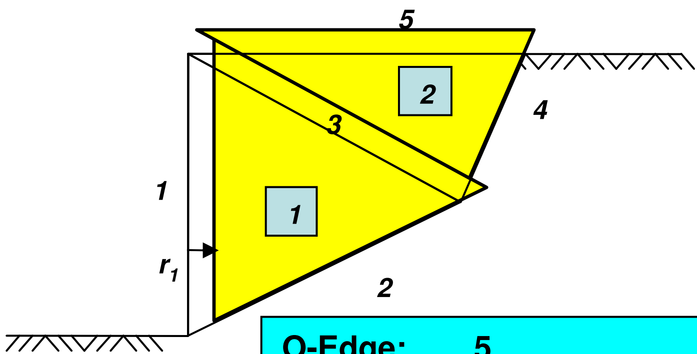
element →

edge ↓

edge	1		2	
1	n_x	n_z	0	0
2	n_x	n_z	0	0
3	n_x	n_z	n_x	n_z
4	0	0	n_x	n_z

$$\cdot \quad v^{sys} = \begin{pmatrix} r_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

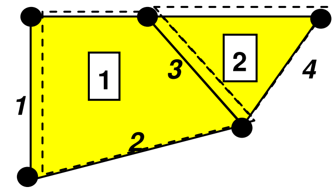
$$K^{sys} \cdot v^{sys} = r^{sys}$$



O-Edge: 5
DR-Edge: *2, 4*
DF-Edge: *1 (gets displ. by r_1 in +dir.)*
I-Edge: *3*

Discretization

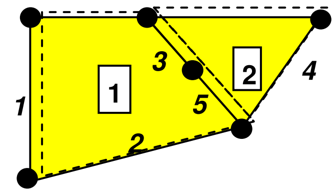
$$K^{sys} \cdot v^{sys} = r^{sys}$$



4 equations
4 unknowns

K: 4 rows × 4 columns • [ ] = [ ]

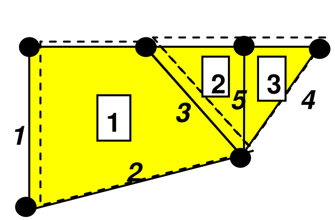
unique solution



5 equations
4 unknowns

K: 5 rows × 4 columns • [ ] = [ ]

residual solution?

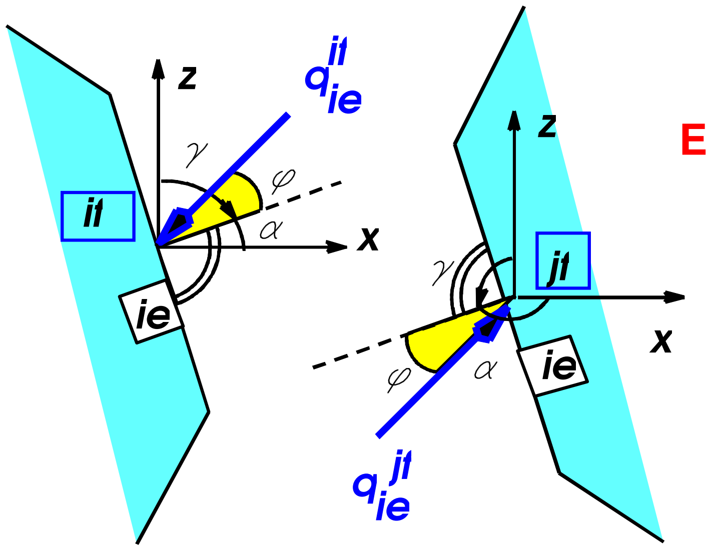


5 equations
6 unknowns

K: 5 rows × 6 columns • [ ] = [ ]

more than one solution

Statics



System:

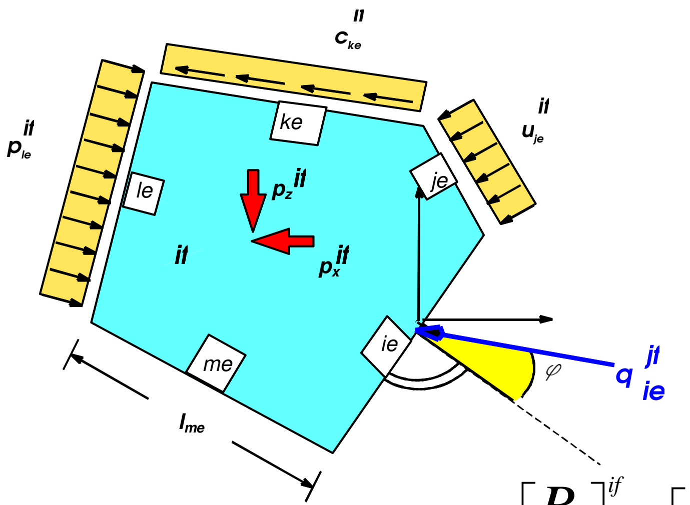
$$F^{sys} = R^{sys} \cdot Q^{sys} + P^{sys}$$

Element:

$$F^{if} = \left[R_{ie} \;\; R_{je} \;\; \ldots R_{ne} \right]^{if} \cdot Q^{if} + p^{if}$$

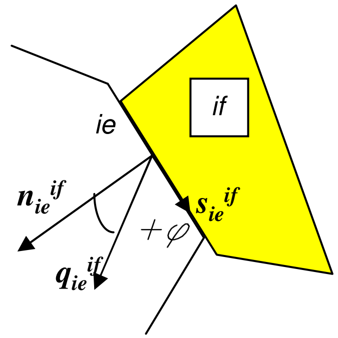
$$\begin{bmatrix} F_x \\ F_z \end{bmatrix}^{if} = \begin{bmatrix} R_{ie_x} & R_{je_x} & \ldots & R_{ne_x} \\ R_{ie_z} & R_{je_z} & \ldots & R_{ne_z} \end{bmatrix}^{if} \cdot \begin{bmatrix} Q_{ie} \\ Q_{je} \\ \cdot \\ \cdot \\ \cdot \\ Q_{ne} \end{bmatrix} + \sum_{i=0}^{i=ne} \begin{bmatrix} p_x^i \\ p_z^i \end{bmatrix}^{if}$$

Equilibrium of forces at one element



$$s_{ie}^{if} = \begin{pmatrix} s_x \\ s_z \end{pmatrix}_{ie}^{if} = T_{ie}^{if\,T} \cdot \tilde{s}_{ie}^{if}$$

$$d_{ie}^{if} = n \times s = \begin{pmatrix} 0 \\ 0 \\ n_x \cdot s_z - n_z \cdot s_x \end{pmatrix}_{ie}^{if}$$



$$\begin{bmatrix} R_x \\ R_z \end{bmatrix}_{ie}^{if} = \begin{bmatrix} \cos\varphi & \chi \cdot \sin\varphi \\ -\chi \cdot \sin\varphi & \cos\varphi \end{bmatrix}_{ie}^{if} \cdot \begin{bmatrix} n_x \\ n_z \end{bmatrix}_{ie}^{if}$$

$$\begin{bmatrix} p_x \\ p_z \end{bmatrix}_{ie}^{if} = \begin{bmatrix} \zeta^* \cdot \cos\gamma \cdot c \cdot l \\ -\zeta^* \cdot \cos\alpha \cdot c \cdot l \end{bmatrix}_{ie}^{if} \cdot \begin{bmatrix} \cos\alpha \cdot (u + p) \cdot l \\ \cos\gamma \cdot (u + p) \cdot l \end{bmatrix}_{ie}^{if}$$

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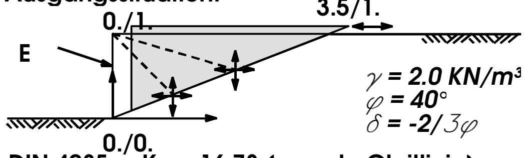
Remarks on discretization level

Ausgangssituation:



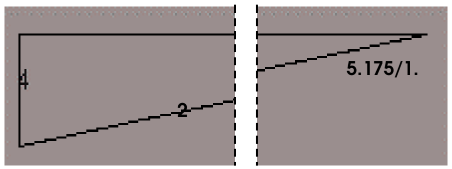
γ = 2.0 KN/m³
φ = 40°
δ = -2/3φ

DIN 4085: K_{ph} = 16.73 (gerade Gleitlinie)
K_{ph} = 11.74 (gekrümmte Gleitlinie)

1 Element: Zielgeometrie:

Kantenkräfte:
Nr.2: 21.542
Nr.4: 18.717 · cos(δ)= 16.726

Optimierungsfreiheitsgrade: 1



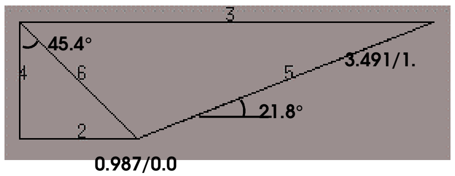
2 Elemente: Zielgeometrie:

Kantenkräfte:
Nr.2: 9.705
Nr.5: 6.352
Nr.4: 13.2729 · cos(δ) =11.861

Optimierungsfreiheitsgrade: 3

vgl. (GUDE) Kap. 2.1.2.4:

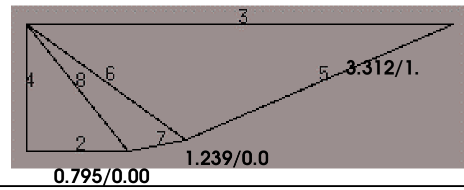
Kpt=13.27· cos(δ) = 11.86, $\vartheta_{1/0}$=0°, $\vartheta_{1/2}$=45°, $\vartheta_{2/0}$=21°



3 Elemente: Zielgeometrie:

Kantenkräfte:
Nr.2: 8.434
Nr 7: 2.747
Nr 5: 4.480
Nr.4: 12.988 · cos(δ)=11.607

Optimierungsfreiheitsgrade: 5

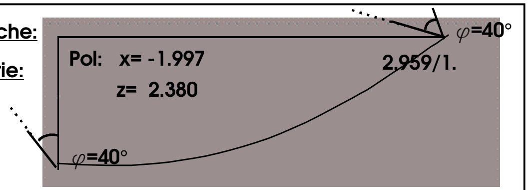


1 Element, spiralförmige Gleitfläche:

Zielgeometrie:

Kantenkraft:
Q = 13.126 · cos(δ) = 11.729

Optimierungsfreiheitsgrade: 1



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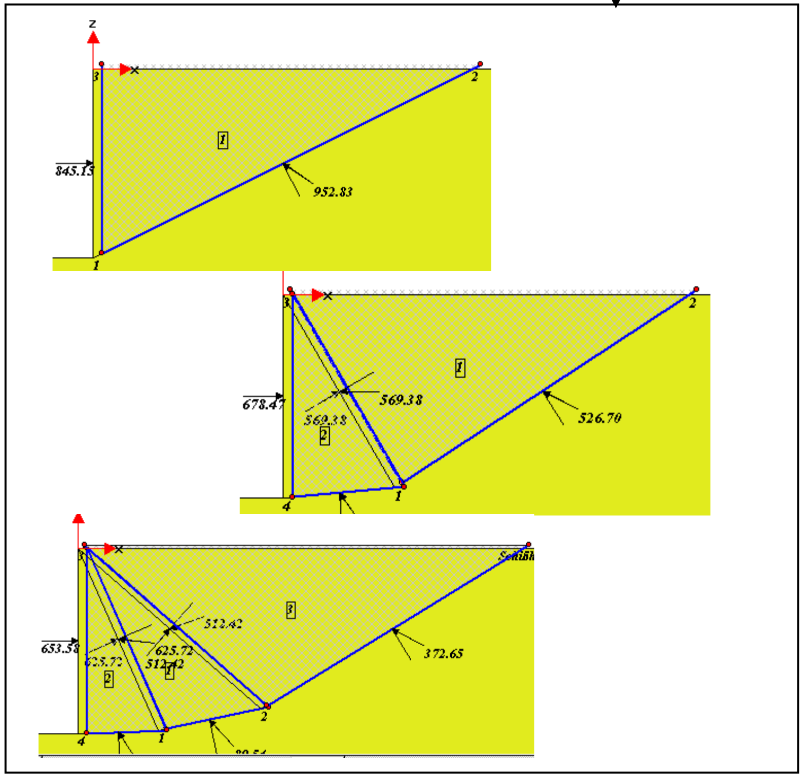
Optimization

Mechanism | **Geoemtry**



Aim ...

- For calculation of ***earth pressure*** the ***remaining force*** of the edge(s) that have predescribed displacement(s) gets optimized (maximized for active and minimized for passive earth pressure).
- For ***internal / external stability*** the aim is defined ***acc. to Fellenius*** (φ, ***c***-reduction for every step during optimization)
- ***Bearing capacity***: Minimum of ***remaining force*** at displ. edges OR safety acc. to ***Fellenius***.

Methods for geometry optimization

Geometry gets changed interactively

Adv.:
- A solution close to the global optimum may be found easily.
- 'Intelligent' way of searching.

Disadv.:
- Not automatically.

Mathematical downhill method (incl. penalty function)

Adv.:
- Fast convergent in most cases.

Disadv.:
- Solution may not be global optimum.

Total Enumeration

Adv.:
- Predefined interval of solution gets "scanned" completely.
- Within this interval the solution fond is a global optimum.

Disadv.:
- Interval of solution has to be defined.
- Calculation takes a long time.

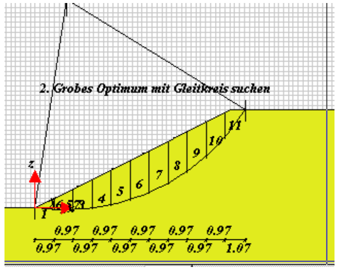
Evolution strategy or genetic Algorithms

Adv.:
- It is pretty likely that the global optimum gets found.

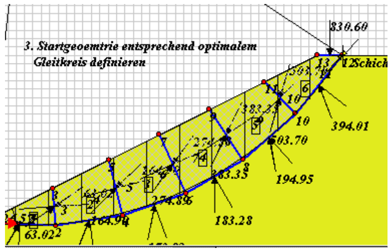
Disadv.:
- Parameters for the algorithm have to be defined properly.
- Calculation takes a long time.

Proposal for geometry optimization

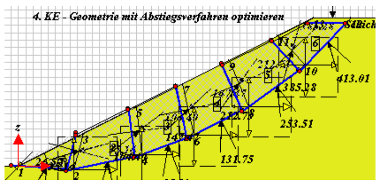
1. Find coarse solution using slipcircle analysis



2. Define start geometry according to slipcicle



3. Optimize by means of math. downhill method



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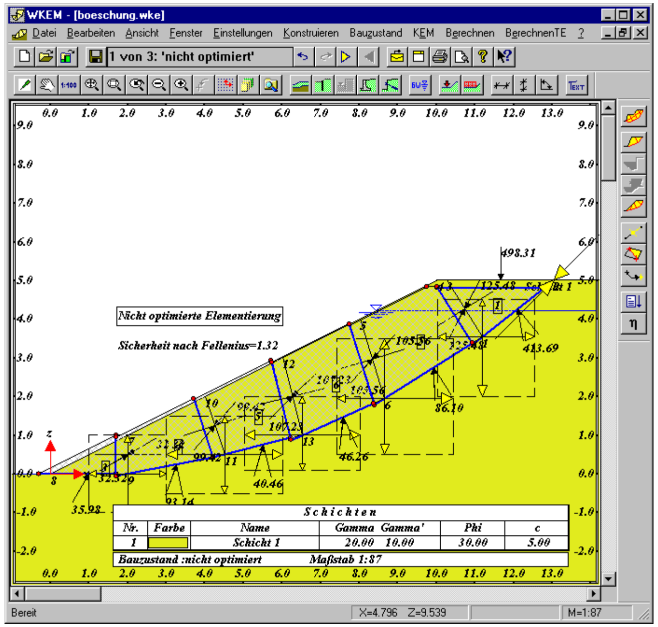
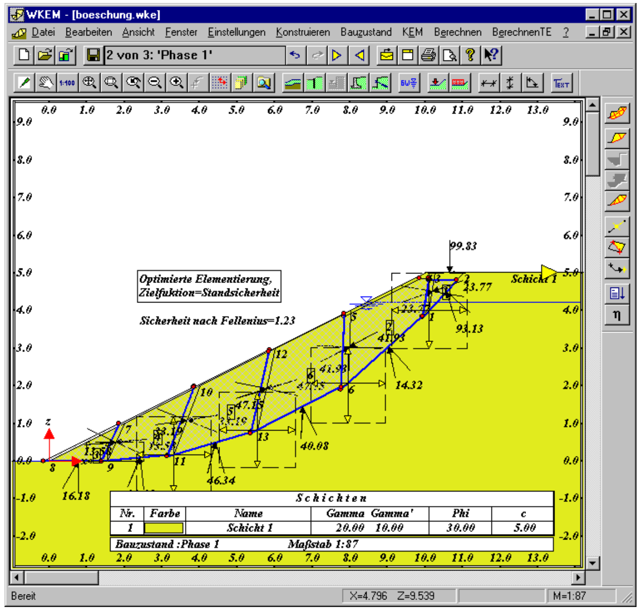
Example: Slope stability

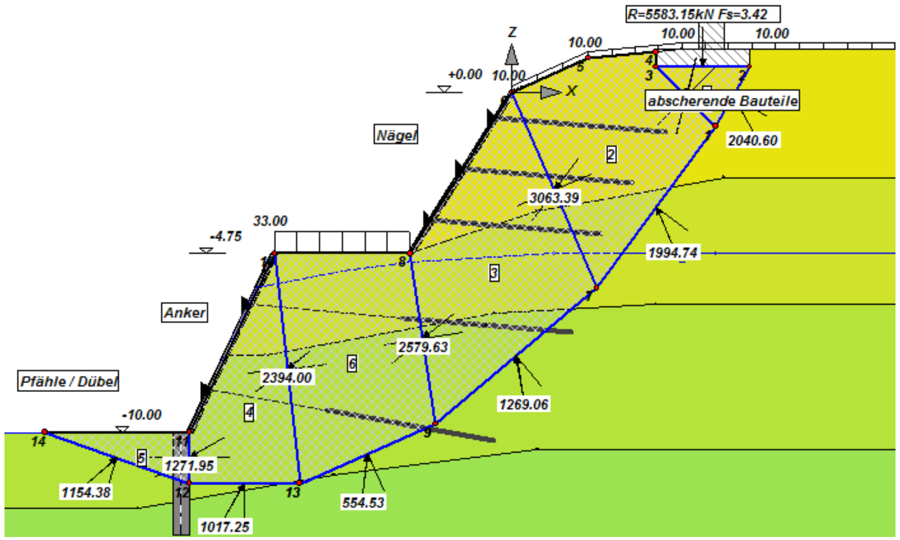
Start mechanism …



best solution found



Construction elements



- Geosynthetics
- Dowels / Piles
- Anchors (prestressed)
- Nails (non prestressed)
- Retainment walls
- Grundwater level(s), and saturation lines
- Construction components (i.e. concrete fundations, …)

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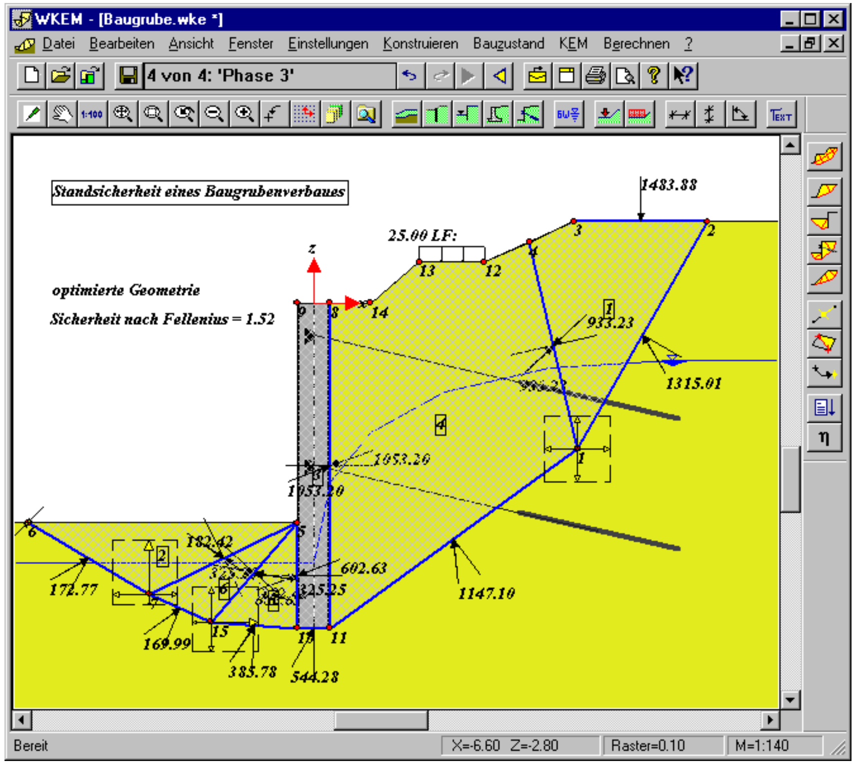
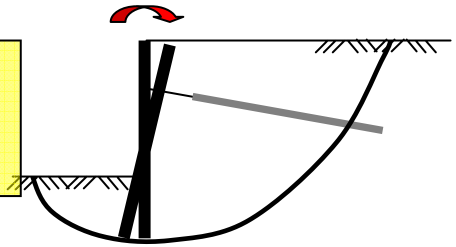
Example: Internal/external Stability of an exc. retainment construction



Potential collapse mechanism ?

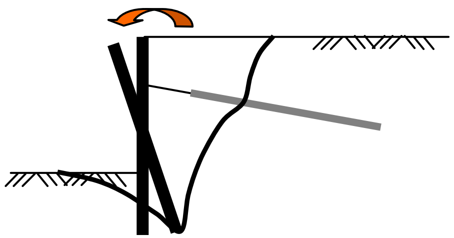
EB 45

„external stability“



EB 44

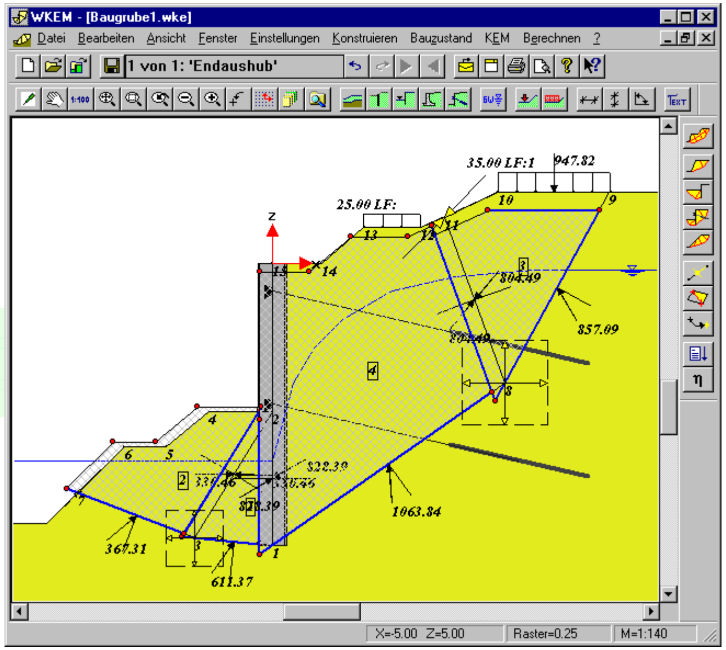
„internal stability“



Example: Internal/external stability of an exc. retainment construction

Advantages …

- No need to distinguish between „internal“ and „external“ stability
- Groundwater: Water pressure and flow gets taken into account element by element correctly
- Earth resistance gets considered automatically (Kinematics!)
- Definition for stability may be defined by user



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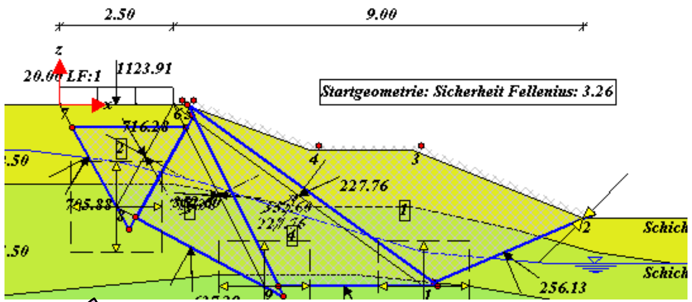
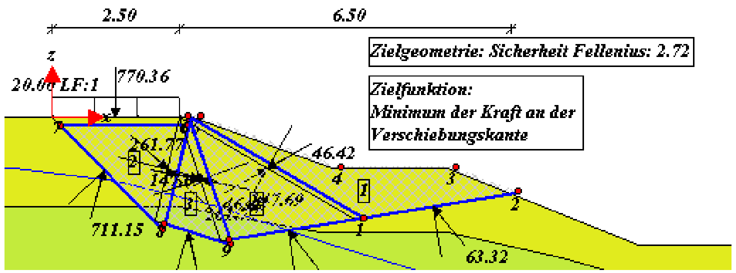
Example: Base failure

Start mechanism:



resulting geometry:



- Arbitrary ground surface
- Arbitrary geometry of soil layers
- Water / flow gets taken into account
- Arbitrary (concentrated) loads

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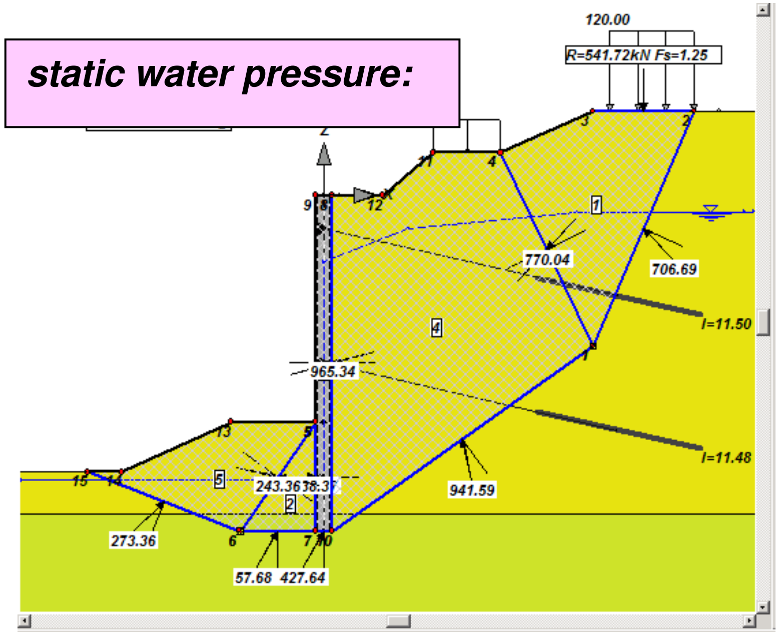
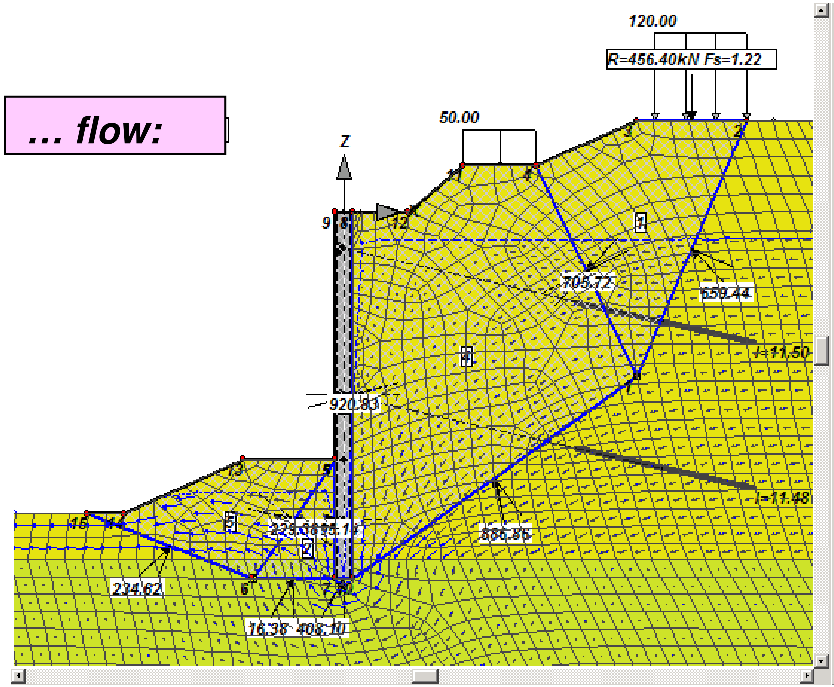
Example: Flow

static water pressure:



… flow:



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